IB Math AA Syllabus

Enhanced representation of the official content syllabus from the IB Subject Guide.

[2019 (first assessment 2021) spec]

SL Content Recommended teaching hours: 120
HL Content Recommended teaching hours: +90

"…" means I was too lazy to copy the full thing from the official IB subject guide.

I **bolded** key words/phrases in the descriptions to make it easier to glance over. I also grouped and named subunits by topic wherever appropriate (Topic column).

Last update: 23rd Apr 2024

#	Topic	Content	Notes
		UNIT 1: Number & Algebra	
1.1	Scientific form	Operations with numbers in **the form $a \times 10^k$** where $1 \le a < 10$ and k is an integer.	Calculator or computer notation is not acceptable. For example, 5.2E30 is not acceptable and should be written as 5.2×10^{30}.
1.2	Arithmetic sequences and series	Use of the formulae for **the n^{th} term** and the **sum of the first n terms** of the sequence. Use of **sigma notation for sums** of sequences.	Spreadsheets, GDCs and graphing software may be used to generate and display sequences in several ways. If technology is used in examinations, students will be expected to identify the first term and the common difference.
		Applications.	**Examples**: simple interest over a number of years.
		Analysis, interpretation, and prediction where a **model is not perfectly arithmetic** in real life.	Students will need to approximate common Differences.
1.3	Geometric sequences and series	Use of the formulae for **the n^{th} term** and the **sum of the first n terms** of the sequence. Use of **sigma notation for sums** of sequences.	If technology is used in examinations, students will be expected to identify the first term and the ratio. **Link to**: models/functions in topic 2 and regression in topic 4.
		Applications.	**Examples**: spread of disease, salary increase and decrease, and population growth.
1.4		**Financial applications** of geometric sequences and series: • **compound interest** • **annual depreciation**.	Examination questions may require the use of technology, including built-in financial packages. Calculate the real value of an investment with an interest rate and an inflation rate. In examinations, questions that ask students to derive the formula will not be set. Compound interest can be calculated yearly, half-yearly, quarterly, or monthly. **Link to**: exponential models/functions in topic 2.
1.5	Integer indices	**Laws of exponents** with integer exponents.	
		Introduction to **logarithms with base 10 and e**. Numerical evaluation of logarithms using technology.	Awareness that $a^x = b$ is equivalent to $\log_a b = x$, that $b > 0$, and $\log_e x = \ln x$.
1.6	Simple deductive proofs	**Simple deductive proof**, numerical and algebraic; how to lay out a left-hand side to right-hand side	**Example**: Show that $\frac{1}{4}+\frac{1}{12}=\frac{1}{3}$. Show that the algebraic generalisation of this is $$\frac{1}{m+1}+\frac{1}{m^2+m}=\frac{1}{m}$$

		(LHS to RHS) proof. The **symbols and notation for equality and identity**.	LHS to RHS proofs require students to begin with the left-hand side expression and transform this using known algebraic steps into the expression on the right-hand side (or vice versa). **Example**: Show that $(x - 3)^2 + 5 \equiv x^2 - 6x + 14$. Students will be expected to show how they can check a result including a check of their own results.
1.7	Logarithms and fractional indices	**Laws of exponents** with rational exponents.	$a^{\frac{1}{m}} = \sqrt[m]{a}$, if m is even this refers to the positive root. **Example**: $16^{\frac{3}{4}} = 8$
		Laws of logarithms. ...	...
		Change of base of a logarithm: $$\log_{\mathrm{a}} x = \frac{\log_{\mathrm{b}} x}{\log_{\mathrm{b}} a}$$	...
		Solving **exponential equations**, including using logarithms.	...
1.8	**Sum of convergent geometric sequences**		Use of $\|r\| < 1$ and modulus notation. **Link to**: geometric sequences and series (1.3).
1.9	Binomial theorem	Expansion of $(\boldsymbol{a} + \boldsymbol{b})^{\boldsymbol{n}}, n \in \mathbb{N}$	Counting principles may be used in the development of the theorem.
		Use of **Pascal's Triangle** **Combinations**; C_r^n	C_r^n should be found using both the formula and technology. **Example**: Find r when $C_r^6 = 20$, using a table of values generated with technology.
1.10 (HL)	Counting principles	**Permutations and Combinations**	**Not required**: Permutations where some objects are identical. Circular arrangements.
		Combinations (C_r^n) with rational n; Extension of the **binomial theorem to fractional and negative indices**: $$(a+b)^n, n \in \mathbb{Q}$$	$$(a+b)^n = \left(a\left(1+\frac{b}{a}\right)\right)^n = a^n\left(1+\frac{b}{a}\right)^n, n \in \mathbb{Q}$$ **Link to**: power series expansions (5.19 HL) **Not required**: Proof of binomial theorem
1.11 (HL)	**Partial fractions**		Maximum of **two** distinct **linear terms** in the denominator, with degree of numerator less than the degree of the denominator. ...
1.12 (HL)	Complex numbers (basics & forms)	$\boldsymbol{i}$ where $i^2 = -1$. Cartesian form $\boldsymbol{z = a + bi}$; the terms **real part**, **imaginary part**, **conjugate**, **modulus**, and **argument**.	
		The **complex plane**.	The complex plane is also known as the Argand diagram. **Link to**: vectors (3.12 HL)
1.13 (HL)		Modulus–argument **(polar) form**: $$z = r(\cos\theta + i\sin\theta) = r \text{ cis } \theta$$ **Euler form**: $$z = re^{i\theta}$$ **Sums, products, and quotients** in Cartesian, polar or Euler forms and their **geometric interpretation**.	The ability to convert between Cartesian, modulus-argument (polar) and Euler form is expected.

1.14 (HL)		**Complex conjugate roots** of quadratic and polynomial equations with real coefficients.	Complex roots occur in conjugate pairs.
		De Moivre's theorem and its extension to rational exponents. **Powers and roots** of complex numbers.	Includes proof by induction for the case where $n \in \mathbb{Z}^+$; awareness that it is true for $n \in \mathbb{R}$ **Link to**: sum and product of roots of polynomial equations (2.12 HL), compound angle identities (3.10 HL).
1.15 (HL)	Proofs	Proof by **Mathematical Induction**.	Proof should be incorporated throughout the course where appropriate. Mathematical induction links specifically to a wide variety of topics, for example complex numbers, differentiation, sums of sequences and divisibility.
		Proof by **contradiction**.	...
		Use of a **counterexample** to show that a statement is not always true.	...
1.16 (HL)	Simultaneous equations	Solutions of **systems of linear equations** (a maximum of three equations in **three unknowns**), including cases where there is a unique solution, an infinite number of solutions or no solution.	...
UNIT 2: Functions			
2.1	Linear functions	Different forms of the **equation of a straight line**. **Gradient**; **intercepts**. Lines with gradients m_1 and m_2 **Parallel** when: $m_1 = m_2$. **Perpendicular** when: $m_1 \times m_2 = -1$.	$y = mx + c$ (gradient-intercept form). $ax + by + d = 0$ (general form). $y - y_1 = m(x - x_1)$ (point-gradient form). Calculate gradients of inclines such as mountain roads, bridges, etc
2.2	Basic functional concepts	Concept of a **function**, **domain**, **range,** and graph. Function **notation**, for example $\mathrm{f}(x)$, $\mathrm{v}(t)$, $\mathrm{C}(n)$. The concept of a function as a **mathematical model**.	...
		Informal concept that an **inverse function** reverses or undoes the effect of a function. Inverse function as a reflection in the line $y = x$, and the notation $\mathrm{f}^{-1}(x)$	...
2.3	Graphing	The **graph of a function**; its equation $y = \mathrm{f}(x)$.	Students should be aware of the difference between the command terms "draw" and "sketch".
		Creating a **sketch** from information given or a context, including transferring a graph from screen to paper. **Using technology** to graph functions including their sums and differences.	All axes and key features should be labelled. This may include functions not specifically mentioned in topic 2.
2.4		Determine **key features** of graphs.	**Maximum** and **minimum** values; **intercepts**; **symmetry**; **vertex**; **zeros** of functions or roots of equations; vertical and horizontal **asymptotes** using graphing technology.

		Finding the **point of intersection** of two curves or lines using technology.	
2.5	Composite functions	**Composite functions**.	$(\mathrm{f} \circ \mathrm{g})(x) = \mathrm{f}(\mathrm{g}(x))$
		Identity function. Finding the inverse function $\mathrm{f}^{-1}(x)$	$(\mathrm{f} \circ \mathrm{f}^{-1})(x) = (\mathrm{f}^{-1} \circ \mathrm{f})(x) = x$ The existence of an inverse for one-to-one functions. **Link to**: concept of inverse function as a reflection in the line $y = x$ (2.2).
2.6	Quadratic functions	The **quadratic function** $f(x) \ = ax^2 + bx + c$ Its graph; y-intercept $(0, c)$; axis of symmetry. **Intercept form** $\mathrm{f}(x) = a(x-p)(x-q)$; x-intercepts $(p, 0)$ and $(q, 0)$. **Vertex form** $\mathrm{f}(x) = a(x-h)^2 + k$; vertex (h, k).	A quadratic graph is also called a parabola. **Link to**: transformations (2.11). Candidates are expected to be able to **change from one form to another**.
2.7		Solution of **quadratic equations** and **inequalities**. The **quadratic formula**.	Using **factorization**, **completing the square** (vertex form), and the **quadratic formula**. Solutions may be referred to as roots or zeros
		The **discriminant** $\boldsymbol{\Delta = b^2 - 4ac}$ and the nature of the roots: two distinct real roots, two equal real roots, and no real roots	...
2.8	Fractional linear functions	The **reciprocal function** $\mathrm{f}(x) = \frac{1}{x}\ \{x \neq 0\}$ Its graph and self-inverse nature.	
		Rational functions of the form $\mathbf{f}(\boldsymbol{x}) = \frac{\boldsymbol{ax+b}}{\boldsymbol{cx+d}}$ and their graphs. **Equations** of vertical and horizontal **asymptotes.**	Sketches should include all horizontal and vertical asymptotes and any intercepts with the axes. **Link to**: transformations (SL2.11). Vertical asymptote: $x = -\frac{d}{c}$ Horizontal asymptote: $y = \frac{a}{c}$
2.9	Exponential functions	**Exponential functions** and their graphs ... **Logarithmic functions** and their graphs ...	... Exponential and logarithmic functions are inverses of each other
2.10	**Solving equations**	Solving equations, both graphically and analytically.	
		Use of technology to solve equations with no valid analytic approach.	
		Application of graphing and solving skills to real-life situations.	**Link to**: exponential growth (2.9)
2.11	Trans-formations	**Transformations** of graphs. Translations, reflections (on axes), horizontal stretch, and vertical stretch.	Students should be aware of the relevance of the order in which transformations are performed. Dynamic graphing packages could be used to investigate these transformations.
		Composite transformations.	Example: Using y = x 2 to sketch y = 3x 2 + 2 **Link to**: composite functions (SL2.5). Not required at SL: transformations of the form $\mathrm{f}(ax + b)$

2.12 (HL)	Polynomial functions	**Polynomial functions**, their graphs, and equations; zeros, roots, and factors. The **factor and remainder theorems**.	
		Sum and **product** of the **roots** of polynomial equations.	The **sum is** $-\frac{a_{n-1}}{a_n}$ The **product is** $(-1)^n \frac{a_0}{a_n}$ **Link to**: complex roots of quadratic and polynomial equations (1.14 HL).
2.13 (HL)	Fractional polynomial functions	**Rational functions** in the form of $$f(x) = \frac{ax+b}{cx^2+dx+e}$$ and $$f(x) = \frac{ax^2+bx+c}{dx+e}$$ Their graphs and **asymptotes**	The reciprocal function is a particular case. Graphs should include all asymptotes (horizontal, vertical and oblique) and any intercepts with axes. Dynamic graphing packages could be used to investigate these functions. **Link to**: rational functions (SL 2.8).
2.14 (HL)	Properties of functions	**Odd** and **even functions**	Even: $f(-x) = f(x)$ Odd: $f(-x) = -f(x)$ Includes periodic functions.
		Finding the **inverse**, including domain restriction	
		Self-inverse functions	
2.15 (HL)	**Functional inequalities**	Solutions of $g(x) \geq f(x)$, Both graphically and analytically.	Graphical or algebraic methods for simple polynomials up to degree 3. Use of technology for these and other functions.
2.16 (HL)	**Modulus functions**	The graphs of the functions $y = \lvert f(x) \rvert$, $y = f(\lvert x \rvert)$, $y = \frac{1}{f(x)}$, $y = f(ax+b)$, and $y = f(x)^2$	Dynamic graphing packages could be used to investigate these transformations.
		Solution of modulus **equations and inequalities**.	**Example**: $\lvert 3x \arccos(x) \rvert > 1$
UNIT 3: Geometry & Trigonometry			
3.1	Basic geometry	The **distance between two points** in three-dimensional space, and their **midpoint**. **Volume** and **surface area** of three-dimensional **solids** including right-pyramid, right cone, sphere, hemisphere and combinations of these solids. The size of an **angle** between two intersecting lines or between a line and a plane.	In SL examinations, only right-angled trigonometry questions will be set in reference to three-dimensional shapes. In problems related to these topics, students should be able to identify relevant right-angled triangles in three-dimensional objects and use them to find unknown lengths and angles.
3.2	Trigonometry I (basics)	Use of **sine**, **cosine**, and **tangent** ratios to find the sides and angles of right-angled triangles.	In all areas of this topic, students should be encouraged to sketch well-labelled diagrams to support their solutions. **Link to**: inverse functions (2.2) when finding angles
		Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ **Cosine rule:** $c^2 = a^2 + b^2 - 2ab \cos C$ **Area:** $\frac{1}{2}ab \sin C$	This section does not include the ambiguous case of the sine rule.

3.3		**Application of** right- and non-right-angled **trigonometry**, including Pythagoras's theorem. Angles of **elevation** and **depression**. **Construction of** labelled **diagrams** from written statements.	Contexts may include use of bearings.
3.4	Angle measure	The circle: **radian** measure of angles; length of an **arc**; area of a **sector**.	Radian measure may be expressed as exact multiples of π, or decimals.
3.5	Standard angles	Definition of $\cos\theta$ and $\sin\theta$ in terms of the **unit circle**.	Includes the relationship between angles in different quadrants. **Examples:** $\cos x = \cos(-x)$ $\tan(3\pi - x) = -\tan x$ $\sin(\pi + x) = -\sin x$
		Definition of **tan** $\boldsymbol{\theta}$ **as** $\frac{\sin\theta}{\cos\theta}$.	...
		Standard angles; Exact values of trigonometric ratios of: $\mathbf{0}, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$, and their multiples.	...
		Extension of the sine rule to the **ambiguous case**.	
3.6	Trigonometry II (identities and equations)	The **Pythagorean Identity** $\cos^2\theta + \sin^2\theta = 1$ **Double angle identities** for sine and cosine.	Simple geometrical diagrams and dynamic graphing packages may be used to illustrate the double angle identities (and other trigonometric identities).
		The relationship between trigonometric ratios.	...
3.7		The **circular functions** $\sin x$, $\cos x$, and $\tan x$; amplitude, their periodic nature, and their graphs. **Composite trig functions** of the form $\mathrm{f}(x) = a\sin(b(x+c)) + d$.	Trigonometric functions may have domains given in degrees or radians. ...
		Transformations.	...
		Real-life contexts.	**Examples**: height of tide, motion of a Ferris wheel.
3.8		**Solving trigonometric equations** in a finite interval, both graphically and analytically.	...
		Equations leading to **quadratics** containing trig functions.	... **Not required**: The general solution of trigonometric equations
3.9 (HL)	Trigonometry III (more functions & identities)	Definition of the **reciprocal trigonometric ratios** $\sec\theta$, $\csc\theta$ and $\cot\theta$. **Pythagorean Identities** $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$ The **inverse functions** $\mathrm{f}(x) = \arcsin x$, etc.; their domains, ranges, and graphs.	
3.10 (HL)		**Compound angle identities**. **Double angle** identity for **tangent**.	Derivation of double angle identities from compound angle identities. **Link to**: De Moivre's theorem (1.14 HL).

3.11 (HL)		**Relationships** between trigonometric functions and the symmetry **properties of their graphs**.	$\sin(\pi-\theta)=\sin\theta$ $\cos(\pi-\theta)=-\cos\theta$ $\tan(\pi-\theta)=-\tan\theta$ **Link to**: the unit circle (SL3.5), odd and even functions (HL2.14), compound angles (HL3.10).
3.12 (HL)	Vectors I (basics)	Concept of a vector; position vectors; displacement vectors. Representation of vectors using **directed line segments**. **Base/unit vectors i, j, k**. Notation: $\boldsymbol{v}=\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}=v_1\mathbf{i}+v_2\mathbf{j}+v_3\mathbf{k}$	
		Algebraic and geometric approaches to **the following**: - sum and difference of vectors - the zero vector, $\mathbf{0}$; the vector $-\boldsymbol{v}$ - multiplication by a scalar, $k\boldsymbol{v}$; parallel vectors - magnitude of a vector, $\lvert\boldsymbol{v}\rvert$ - unit vectors, $\frac{\boldsymbol{v}}{\lvert\boldsymbol{v}\rvert}$ - position vectors: $\overrightarrow{\mathrm{OA}}=\boldsymbol{a}$, $\overrightarrow{\mathrm{OB}}=\boldsymbol{b}$ - displacement vectors; $\overrightarrow{\mathrm{AB}}=\boldsymbol{b}-\boldsymbol{a}$ **Proofs of geometrical properties** using vectors.	
3.13 (HL)		The **scalar/dot product** of two vectors. The **angle between two vectors**. **Perpendicular** vectors; **parallel** vectors.	Applications of the properties of the dot product $\boldsymbol{v}\cdot\boldsymbol{w}=\boldsymbol{w}\cdot\boldsymbol{v}$; $\boldsymbol{u}\cdot(\boldsymbol{v}+\boldsymbol{w})=\boldsymbol{u}\cdot\boldsymbol{v}+\boldsymbol{u}\cdot\boldsymbol{w}$; $(k\boldsymbol{v})\cdot\boldsymbol{w}=k(\boldsymbol{v}\cdot\boldsymbol{w})$; $\boldsymbol{v}\cdot\boldsymbol{v}=\lvert\boldsymbol{v}\rvert^2$. $\boldsymbol{v}\cdot\boldsymbol{w}=\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$, where θ is the **angle** between $\boldsymbol{v}$ and $\boldsymbol{w}$. For non-zero vectors, $\boldsymbol{v}\cdot\boldsymbol{w}=0$ is equivalent to the vectors being **perpendicular**. For **parallel** vectors, $\lvert\boldsymbol{v}\cdot\boldsymbol{w}\rvert=\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert$.
3.14 (HL)	Vector linear equations	**Vector equation of a line** in two and three dimensions: $\boldsymbol{r}=\lambda\boldsymbol{b}+\boldsymbol{a}$ **Parametric** form: $x=x_0+\lambda b_x$ $y=y_0+\lambda b_y$ $z=z_0+\lambda b_z$ **Cartesian** form: $\frac{x-x_0}{b_x}=\frac{y-y_0}{b_y}=\frac{z-z_0}{b_z}$	Relevance of $\boldsymbol{a}$ (initial position) and $\boldsymbol{b}$ (direction).
		The angle between two lines.	Using the scalar product of the two direction vectors.
		Simple applications to kinematics.	Interpretation of λ as time and $\boldsymbol{b}$ as velocity, with $\lvert\boldsymbol{b}\rvert$ representing speed.
3.15 (HL)		Coincident, parallel, intersecting, and skew lines, distinguishing between these cases. Points of intersection.	Skew lines are non-parallel lines that do not intersect in three-dimensional space.
3.16		Vector/cross product.	"Vector product" = "Cross product".

(HL)	Vectors II (cross product)		$\boldsymbol{v} \times \boldsymbol{w} = (\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert \sin\theta)\boldsymbol{n}$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$, and $\boldsymbol{n}$ is the unit normal vector whose direction is given by the **right hand screw rule**.
		Properties of the cross product.	$\boldsymbol{v} \times \boldsymbol{w} = -\boldsymbol{w} \times \boldsymbol{v}$; $\boldsymbol{u} \times (\boldsymbol{v} + \boldsymbol{w}) = \boldsymbol{u} \times \boldsymbol{v} + \boldsymbol{u} \times \boldsymbol{w}$; $(k\boldsymbol{v}) \times \boldsymbol{w} = k(\boldsymbol{v} \times \boldsymbol{w})$; $\boldsymbol{v} \times \boldsymbol{v} = \boldsymbol{0}$. For non-zero vectors, $\boldsymbol{v} \times \boldsymbol{w} = \boldsymbol{0}$ is equivalent to the vectors being **parallel**.
		Geometric interpretation of $\lvert\boldsymbol{v} \times \boldsymbol{w}\rvert$.	Use of $\lvert\boldsymbol{v} \times \boldsymbol{w}\rvert$ to **find** the **area of a parallelogram** (and hence a triangle).
3.17 (HL)	Planes (not birds)	**Vector equations of a plane**: $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$ where $\boldsymbol{b}$ and $\boldsymbol{c}$ are non-parallel vectors within the plane. $\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$ where $\boldsymbol{n}$ is a normal to the plane and $\boldsymbol{a}$ is the position vector of a point on the plane. **Cartesian** form: $ax + by + cz = d$	
3.18 (HL)		**Intersections** of: a line with a plane; two planes; three planes. **Angle between**: a line and a plane; two planes.	Finding intersections by solving equations; geometrical interpretation of solutions. **Link to**: solutions of systems of linear equations (HL 1.16).
UNIT 4: Statistics & Probability			
4.1	Sampling	**Concepts** of population, sample, random sample, discrete and continuous data.	This is designed to cover the key questions that students should ask when they see a data set/ analysis.
		Reliability of data sources and **bias** in sampling.	Dealing with missing data, errors in the recording of data.
		Interpretation of **outliers**.	Outlier is defined as a data item which is more than $1.5 \times$ interquartile range (IQR) from the nearest quartile. Awareness that, in context, some outliers are a valid part of the sample, but some outlying data items may be an error in the sample. **Link to**: box and whisker diagrams (4.2) and measures of dispersion (4.3).
		Sampling techniques and their effectiveness.	Simple random, convenience, systematic, quota and stratified sampling methods.
4.2	Statistical graphs	**Presentation of data** (discrete and continuous): frequency distributions (tables).	Class intervals will be given as inequalities, without gaps.
		Histograms. **Cumulative frequency**; cumulative frequency graphs; use to find median, quartiles, percentiles, range, and interquartile range (IQR).	Frequency histograms with equal class intervals. **Not required**: Frequency density histograms.
		Production and understanding of **box and whisker diagrams**.	Use of box and whisker diagrams to compare two distributions, using symmetry, median, interquartile

			range, or range. Outliers should be indicated with a cross. Determining whether the data may be normally distributed by consideration of the symmetry of the box and whiskers.
4.3	Statistics	**Measures of central tendency** (mean, median and mode). **Estimation of mean** from grouped data.	Calculation of mean using formula and technology. Students should use mid-interval values to estimate the mean of grouped data.
		Modal class.	For equal class intervals only.
		Measures of dispersion (interquartile range, standard deviation, and variance).	Calculation of standard deviation and variance of the sample using only technology; however, hand calculations may enhance understanding. Variance is the square of the standard deviation.
		Effect of constant changes on the original data.	**Examples**: If three is subtracted from the data items, then the mean is decreased by three, but the standard deviation is unchanged. If all the data items are doubled, the mean is doubled, and the standard deviation is also doubled.
		Quartiles of discrete data.	Using technology. Awareness that different methods for finding quartiles exist and therefore the values obtained using technology and by hand may differ.
4.4	Bivariate analysis	**Linear correlation** of bivariate data. **Pearson's product-moment correlation coefficient**, r.	Technology should be used to calculate r. However, hand calculations of r may enhance understanding. Critical values of r will be given where appropriate. Students should be aware that Pearson's product moment correlation coefficient (r) is only meaningful for linear relationships.
		Scatter diagrams; **lines of best fit**, by eye, passing through the mean point.	Positive, zero, negative; strong, weak, no correlation. Students should be able to make the distinction between correlation and causation and know that correlation does not imply causation.
		Equation of the **regression line** of y on x. Use of the equation of the **regression line for prediction** purposes. Interpret the **meaning of the parameters, a and b**, in a linear regression $y = ax + b$	Technology should be used to find the equation. Students should be aware: • of the dangers of extrapolation • that they cannot always reliably make a prediction of x from a value of y, when using a y on x line.
4.5	Probability basics	**Concepts** of trial, outcome, equally likely outcomes, relative frequency, sample space (U) and event. The probability of an event A is $\boldsymbol{P(A) = \frac{n(A)}{n(U)}}$. The **complementary events** A and A' (not-A).	Sample spaces can be represented in many ways, for example as a table or a list. Experiments using coins, dice, cards and so on, can enhance understanding of the distinction between experimental (relative frequency) and theoretical probability. Simulations may be used to enhance this topic.
		Expected number of **occurrences**.	**Example**: If there are 128 students in a class and the probability of being absent is 0.1, the expected number of absent students is 12.8.

4.6	Probability calculations	Use of **Venn diagrams**, **tree diagrams**, **sample space diagrams** and **tables** of outcomes to calculate probabilities.	
		Combined events: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. **Mutually exclusive events**: $\mathrm{P}(A \cap B) = 0$.	The non-exclusivity of "or".
		Conditional probability: $\mathrm{P}(A\|B) = \mathrm{P}(A \cap B)\ \mathrm{P}(B)$.	An alternate form of this is: $\mathrm{P}(A \cap B) = \mathrm{P}(B)\mathrm{P}(A\|B)$. Problems can be solved with the aid of a Venn diagram, tree diagram, sample space diagram or table of outcomes without explicit use of formulae. Probabilities with and without replacement.
		Independent events: $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$.	
4.7	Discrete random variables I (basics)	Concept of **discrete random variables** and their probability distributions. **Expected value** (mean), for discrete data. Applications.	...
4.8	Binomial distribution	**Binomial distribution**. **Mean and variance** of the binomial distribution.	Situations where the binomial distribution is an appropriate model. In examinations, binomial probabilities should be found using available technology. **Not required**: Formal proof of mean and variance. **Link to**: expected number of occurrences (4.5).
4.9	Normal distribution	The **normal distribution** and curve. Properties of the normal distribution. Diagrammatic **representation**.	Awareness of the natural occurrence of the normal distribution. Students should be aware that approximately 68% of the data lies between $\mu \pm \sigma$, 95% lies between $\mu \pm 2\sigma$ and 99.7% of the data lies between $\mu \pm 3\sigma$.
		Normal probability **calculations**.	Probabilities and values of the variable must be found using technology
		Inverse normal **calculations**	For inverse normal calculations mean and standard deviation will be given. This does not involve transformation to the standardized normal variable z.
4.10	Linear regression	**Equation** of the regression line of $\boldsymbol{x}$ **on** $\boldsymbol{y}$.	
		Use of the **equation for prediction** purposes.	Students should be aware that they cannot always reliably make a prediction of y from a value of x, when using an x on y line.
4.11	Conditional probabilities	Formal definition and use of the formulae $\mathrm{P}(A\|B) = \frac{\mathrm{P}(A \cap B)}{P(B)}$ for **conditional probabilities**, and $\mathrm{P}(A\|B) = \mathrm{P}(A) = \mathrm{P}(A\|B')$ for **independent events**.	An alternate form of this is: $\mathrm{P}(A \cap B) = \mathrm{P}(B)\mathrm{P}(A\|B)$. Testing for independence.
4.12		Standardization of normal variables (**$\boldsymbol{z}$-values**).	Probabilities and values of the variable must be found using technology.

	Standardized normal variables		The standardized value (z) gives the number of standard deviations from the mean.
		Inverse normal calculations where **mean** and **standard deviation** are **unknown**.	Use of z-values to calculate unknown means and standard deviations.
4.13 (HL)	Bayes' theorem	Use of **Bayes' theorem** for a maximum of three events.	**Link to**: independent events (4.6)
4.14 (HL)	Discrete random variables II (statistics)	**Variance** of a discrete random variable.	**Link to**: discrete random variables (4.7)
		Continuous random variables and their **probability density functions**.	...
		Mode and median of continuous random variables.	...
		Mean, variance and **standard deviation** of both discrete and continuous random variables.	...
		The effect of **linear transformations** of X.	...
UNIT 5: Calculus			
5.1	Calculus fundamentals	Introduction to the concept of a **limit**.	Estimation of the value of a limit from a table or graph.
		Derivative interpreted as gradient function and as rate of change.	Forms of notation ...
5.2		**Increasing and decreasing** functions. Graphical interpretation of $f'(x) > 0$, $f'(x) = 0$, $f'(x) > 0$	Identifying intervals on which functions are increasing or decreasing.
5.3	Basic differentiation	**Power rule** (integer powers)**:** Derivative of ax^n is anx^{n-1}. Derivatives of (Laurent) polynomials.	
5.4		**Tangents** and **normals** at a given point, and their **equations**.	Use of both analytic approaches and technology.
5.5	Basic integration	Introduction to **integration** as anti-differentiation... Of (Laurent) polynomials ($n \neq -1$).	
		Anti-differentiation with a boundary condition to determine the **constant term**.	**Example**: if $\frac{dy}{dx} = 3x^2 + x$ and $y = 10$ when $x = 1$, then $y = x^3 + \frac{1}{2}x^2 + 8.5$.
		Definite integrals using technology. **Area** of a region **enclosed by a curve** $y = f(x)$ and the x-axis, where $\mathbf{f(x) > 0}$.	Students are expected to first write a correct expression before calculating the area, ... The use of dynamic geometry or graphing software is encouraged in the development of this concept.
5.6	Analytical differentiation methods	**Power rule** for $n \in \mathbb{Q}$ **Derivatives of $\sin x$, $\cos x$, and $\ln x$.** Differentiation of sums and multiples of these functions.	
		Chain rule.	**Example**: ...
		Product and quotient rules.	**Link to**: composite functions (SL2.5).
5.7	Applications of derivatives	The **second derivative**.	...

		Graphical behaviour of functions, including the relationship between the graphs of f,f' and f''.	
5.8		Local **maximum and minimum** points. **Testing** for maximum and minimum.	...
		Optimization.	Examples of optimization may include profit, area, and volume.
		Points of **inflexion** with zero and non-zero gradients.	At a point of inflexion, $\mathrm{f}''(x) = 0$ **and** changes sign (concavity change). ...
5.9	Kinematics	**Kinematic problems** involving displacement s, velocity v, acceleration a, and total distance travelled.	$v = \frac{\mathrm{d}s}{\mathrm{d}t}$; $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ Displacement from t_1 to t_2 is given by $\int_{t_1}^{t_2} \mathrm{v}(t)\mathrm{d}t$. Distance between t_1 to t_2 is given by $\int_{t_1}^{t_2} \lvert\mathrm{v}(t)\rvert\mathrm{d}t$. Speed is the magnitude of velocity.
5.10	Analytical integration methods	**Reverse-power rule** for $n \in \mathbb{Q}$. **Indefinite integrals of $\sin x$, $\cos x$, $\frac{1}{x}$, and e^x.**	$\int \frac{1}{x}\mathrm{d}x = \ln\lvert x\rvert + C$
		The **composites** of any of these with the **linear function** $ax + b$.	**Example**: ...
		Integration by inspection (**reverse chain rule**) or **by substitution** for expressions of the form: $$\int k \times \mathrm{f}(\mathrm{g}(x))\mathrm{g}'(x)\,\mathrm{d}x$$	**Examples**: ...
5.11	Definite integrals	**Definite integrals**, including analytical approach.	...
		Areas of a region **enclosed by a curve** $y = \mathrm{f}(x)$ and the x-axis, where $\mathrm{f}(x)$ can be positive or negative, without the use of technology. **Areas between curves.**	
5.12 (HL)	Limits and differentiation	Informal understanding of continuity and differentiability of a function at a point.	In examinations, students will not be asked to test for continuity and differentiability.
		Understanding of limits (convergence and divergence). **Definition of derivative** from first principles ...	**Link to**: infinite geometric sequences (SL1.8). Use of this definition for polynomials only.
		Higher derivatives.	Familiarity with the **notations $\frac{d^n y}{dx^n}$** and $\mathbf{f}^{(n)}(x)$. **Link to**: proof by mathematical induction (1.15HL).
5.13 (HL)	**L'Hôpital's rule**	Evaluation of limits of the form $\lim_{x\to a} \frac{\mathrm{f}(x)}{\mathrm{g}(x)}$ and $\lim_{x\to\infty} \frac{\mathrm{f}(x)}{\mathrm{g}(x)}$ using l'Hôpital's rule or the Maclaurin series.	The indeterminate forms $\frac{0}{0}$ and $\frac{\infty}{\infty}$. **Example**: $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$. **Link to**: horizontal asymptotes (SL2.8).
		Repeated use of l'Hôpital's rule.	
5.14 (HL)	Related rates	**Implicit differentiation.** **Related rates** of change. **Optimisation problems.**	Appropriate use of the chain rule or implicit differentiation, including cases where the optimum solution is at the end point.

5.15 (HL)	Advanced analytical calculus	**Derivatives** of all **trig** and **exponential functions** and their **inverses**.	$\tan x$, $\sec x$, $\csc x$, $\cot x$, a^x, $\log_a x$, $\arcsin x$, $\arccos x$, and $\arctan x$.
		Indefinite integrals of the derivatives of any of the **above functions**. The **composites** of any of these with a **linear function**.	Indefinite integral interpreted as a family of curves. **Examples**: $\int \frac{1}{x^2+2x+5}\,\mathrm{d}x = \frac{1}{2}\arctan\frac{x+1}{2} + C$ $\int \sec^2(2x+5)\,\mathrm{d}x = \frac{1}{2}\tan(2x+5) + C$
		Use of **partial fractions to rearrange** the integrand.	$\int \frac{1}{x^2+3x+2}\mathrm{d}x = \ln\left\lvert\frac{x+1}{x+2}\right\rvert + C$ **Link to**: partial fractions (HL1.11)
5.16 (HL)		**Integration by substitution.**	On examination papers, substitutions will be provided if the integral is not of the form $\int k \times \mathrm{f}(\mathrm{g}(x))\mathrm{g}'(x)\,\mathrm{d}x$. **Link to**: integration by substitution (SL5.10).
		Integration by parts.	...
		Repeated integration by parts.	...
5.17 (HL)	Area and volume	Area of the region enclosed by a curve and the y axis in a given interval. **Volumes of revolution** about the x-axis or y-axis.	
5.18 (HL)	Differential equations	First order differential equations (**1st order linear ODE**). Numerical solution of $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x,y)$ using **Euler's method**.	$x_{x+1} = x_n + h$, where h is a constant.
		Separable differential equations. (Solving them.)	**Example**: the logistic equation $\frac{\mathrm{d}n}{\mathrm{d}t} = kn(a-n), \qquad a,k \in \mathbb{R}$ **Link to**: partial fractions (HL1.11) and use of partial fractions to rearrange the integrand (HL5.15).
		Homogeneous differential equations; Solving 1st order linear ODEs of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(\frac{y}{x})$ using the substitution $y = vx$.	
		Integrating factor method; Solving 1st order linear ODEs of the form $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$...	...using an "integrating factor" $\mathrm{I}(x) = e^{\int \mathrm{P}(x)\mathrm{d}x}$. For certain $\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{P}(x)y = \mathrm{Q}(x)$, $y = e^{-\int \mathrm{P}(x)\mathrm{d}x} \int e^{\int \mathrm{P}(x)\mathrm{d}x}\mathrm{Q}(x)\,\mathrm{d}x$.
5.19 (HL)	**Maclaurin series**	Maclaurin series to obtain expansions for e^x, $\sin x$, $\cos x$, and $\ln(1+x)^p$, $p \in \mathbb{Q}$.	
		Use of simple substitution, products, integration, and differentiation to obtain other series.	**Example**: for substitution: replace x with x^2 to define the Maclaurin series for e^{x^2}. **Example**: the expansion of $e^x \sin x$.
		Maclaurin series developed from differential equations.	
END OF SYLLABUS :)			
#	**Topic**	**Content**	**Notes**